Appendix

1 Payoffs to each strategy from playing the public goods game

Let $V(A|i,j)$ be the payoff to an individual of strategy type A from playing a public goods game in a social group where there are i *Cooperators* (C), j *Defectors* (D) and $l = n-1-i-j$ *Loners* (L), among the $n-1$ others in the group.

1.1 Non-rival consumption

The payoffs if consumption of the public goods is non-rival are:

$$\begin{aligned} V(L|i,j) &= h_l \\ V(D|i,j) &= \begin{cases} h_d + \frac{i}{n}b & \text{if } i+j>0 \\ h_l & \text{if } i+j<0 \end{cases} \\ V(C|i,j,k) &= \begin{cases} h_c + \frac{i+1}{n}b & \text{if } i+j>0 \\ h_l & \text{if } i+j<0 \end{cases} \end{aligned}$$

1.2 Rival consumption

The payoffs if consumption of the public goods is rival are:

$$\begin{aligned} V(L|i,j) &= h_l \\ V(D|i,j) &= \begin{cases} h_d + \frac{\left(\frac{i}{n}\right)b + \left(\frac{i}{n}\right)^2 s}{i+j+1} & \text{if } i+j>0 \\ h_l & \text{if } i+j=0 \end{cases} \\ V(C|i,j) &= \begin{cases} h_c + \frac{\left(\frac{i+1}{n}\right)b + \left(\frac{i+1}{n}\right)^2 s}{i+j+1} & \text{if } i+j>0 \\ h_l & \text{if } i+j=0 \end{cases} \end{aligned}$$

2 Expected payoffs

To calculate the fitness of each type we derive the expected payoff of each strategy given by P_L, P_D, and P_C . The probability that a focal individual being in a group with i *Cooperators*, j *Defectors* and $l = n-1-i-j$ *Loners* among the other $n-1$ individuals, $\Pr(i,j)$, is

$$\Pr(i,j) = \frac{(n-1)!x^i y^j z^l}{i!j!l!}$$

where
x = frequency of *Cooperators* in the population
y = frequency of *Defectors* in the population
z = frequency of *Loners* in the population
and $x + y + z = 1$.

2.1 Non-rival consumption

$$\begin{aligned}
P_L &= h_l \\
P_D &= p(i+j>0)E[h_d + \frac{i}{n}b|i+j>0] + p(i+j=0)h_l \\
&= h_d(1-z^{n-1}) + \frac{b}{n}x(n-1) + h_l z^{n-1} \\
P_C &= p(i+j>0)E[h_c + \frac{i+1}{n}b|i+j>0] + p(i+j=0)h_l \\
&= (h_c + \frac{b}{n})(1-z^{n-1}) + \frac{b}{n}x(n-1) + h_l z^{n-1}
\end{aligned}$$

2.2 Rival consumption

$$
\begin{aligned}
P_L &= h_l \\
P_D &= p(i+j>0)E\left(h_d + \frac{b\left(\frac{i}{n}\right)+s\left(\frac{i}{n}\right)^2}{1+i+j}\middle| i+j>0\right) + p(i+j=0)h_l \\
&= h_d(1-z^{n-1}) + F(x,z)\left(\frac{b}{n}+\frac{s}{n^2}\right) - \left(\frac{x}{1-z}\right)F(x,z)\frac{2s}{n^2} \\
&\quad + \frac{s}{n^2}\left(\frac{x}{1-z}\right)^2(1-z)(n-1) + h_l z^{n-1} \\
P_C &= p(i+j>0)E\left(h_c + \frac{b\left(\frac{i+1}{n}\right)+s\left(\frac{i+1}{n}\right)^2}{1+i+j}\middle| i+j>0\right) + p(i+j=0)h_l \\
&= h_c(1-z^{n-1}) + F(x,z)\left(\frac{b}{n}+\frac{3s}{n^2}\right) - \left(\frac{x}{1-z}\right)F(x,z)\frac{2s}{n^2} \\
&\quad + \frac{s}{n^2}\left(\frac{x}{1-z}\right)^2(1-z)(n-1) + \left(\frac{1-z^n}{n(1-z)}\right)\left(\frac{b}{n}+\frac{s}{n^2}\right) + \left(h_l - \frac{b}{n} - \frac{s}{n^2}\right)z^{n-1}
\end{aligned}
$$

where
$F\left(x,z\right) = \left(\frac{x}{1-z}\right)\left(1-\frac{1-z^n}{n(1-z)}\right)$